

Geiger-Müller Detector

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Phys. 2033: Quantum Lab

1 Purpose

The purpose of this lab is to exercise statistical methods in analyzing the randomness inherent with radiation. Specifically, we measured the radioactive properties of a sample, the background radiation in the lab environment, and the dead-time of the equipment being used. Looking at these three data sets together, we drew conclusions about error.

2 Methodology

Using the equipment described below we collected three sets of data. Each data set is explained in detail in the *Collected Data* section. Our overall method involved counting radioactive events in a controlled laboratory environment, and using those counts for later analysis.

2.1 Equipment Used

For this lab, we used a Geiger-Müller detector connected to a unified power supply and counting apparatus. As shown in *Fig. 1*, the detector is mounted above a set of shelves and surrounded on three sides by lead shielding. Samples are placed onto the shown holder tray and inserted into a specific shelf before counting begins.

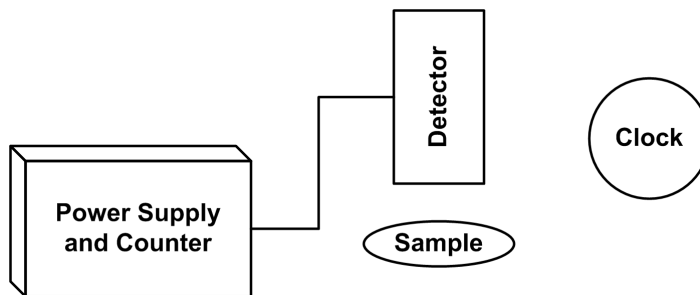


Figure 1:

The power apparatus applies a user-defined variable voltage to the detector. Any events caught by the detector are then returned to the apparatus for counting. The apparatus is designed to count only for a user-defined period of time.

2.2 Physics Demonstrated

We are specifically working with radioactive materials, which release α , β , or γ rays at random intervals. As the particles are released from the sample and travel through the detector, they trigger a cascade that can cause the voltage to short across the chamber. These shorts can then be counted.

In addition, this randomness requires us to apply several statistical methods in order to correctly analyze the samples we use. By taking multiple measurements of the same conditions, we can attempt to measure and correct for the inherent randomness.

3 Collected Data

Below we list all data collected as we progressed through the lab. The method used to collect data in *Tab. 2* and *Tab. 3* is based on results obtained from *Tab. 1*, as explained in the *Analysis and Results* section.

Voltage (V)	Count (N)	Voltage (V)	Count (N)
0	0	550	172
100	0	600	169
200	0	650	182
300	0	700	159
325	146	750	171
350	156	800	154
400	192	850	184
450	185	900	219
500	175		

Table 1: Measured response from a radioactive sample when voltage V is applied to the detector. Time interval is 1 minute and conditions remain unchanged through all measurements.

Trial	Count (N)	Trial	Count (N)	Trial	Count (N)	Trial	Count (N)
1	5	14	6	27	2	40	7
2	6	15	7	28	10	41	5
3	3	16	4	29	6	42	4
4	5	17	3	30	8	43	14
5	8	18	3	31	6	44	4
6	3	19	3	32	8	45	6
7	8	20	9	33	3	46	5
8	6	21	6	34	4	47	5
9	7	22	8	35	4	48	2
10	3	23	5	36	4	49	3
11	4	24	5	37	3	50	4
12	3	25	8	38	4		
13	3	26	6	39	5		

Table 2: Detected background radiation in lab environment. Time interval is 30 seconds and conditions remain unchanged through all measurements.

Trial	N	$(N - \bar{N})^2$	Trial	N	$(N - \bar{N})^2$	Trial	N	$(N - \bar{N})^2$
1	338	196	10	347	25	19	338	196
2	352	0	11	334	324	20	353	1
3	334	324	12	374	484	21	344	64
4	370	324	13	368	256	22	355	9
5	384	1024	14	394	1764	23	354	4
6	339	169	15	383	961	24	353	1
7	316	1296	16	323	841	25	335	289
8	359	49	17	351	1			
9	367	225	18	331	441			

Table 3: Detected response from radioactive source when $V = 500$ volts. Time interval is 1 minute and conditions remain unchanged through all measurements.

Test Case	Count (N)	$\sigma = \sqrt{N}/T$	Corrected (r_{back})
(1) Sample A only	101386	2.21%	98707
(2) Sample B only	7563	0.60%	4884
(3) Both samples	108336	2.29%	105657

Table 4: Measured response from given sample configuration over a period of 4 hours.

4 Analysis and Results

As mentioned earlier, the lab consisted of four major parts.

4.1 Geiger Plateau

First, we determined an appropriate operating voltage for the detector. We did this by measuring the response of a ^{60}Co source as we varied voltage. All count measurements were taken using a time interval of 30 seconds and all conditions remained unchanged through the entire process. The raw data results are shown in *Tab. 1*. We found the threshold voltage (where the detector began counting events) to be $V = 320$ volts.

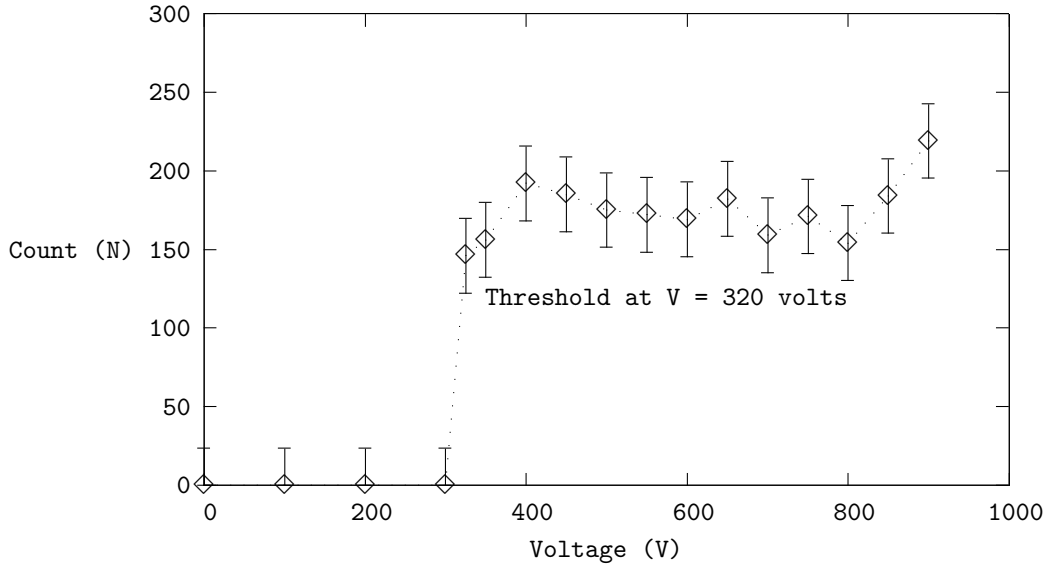


Figure 2: Plot of radiation events (Count) as voltage is varied. Time interval is 30 seconds and conditions remain unchanged through all measurements. The errorbars illustrate two standard deviations.

Using the average count from all active counts ($\bar{N} = 174$), we calculated a standard deviation of $\sigma = 11.88$. We then graphed the detector response as a function of applied voltage (*Fig. 1*). The included errorbars illustrate two standard deviations as calculated above. Using this information, we chose an operating voltage of $V = 500$ volts to be used for the two remaining parts of the lab.

4.2 Background Radiation

Second, we measured the natural radiation present in the lab environment. We did this by clearing the area of all radioactive samples and counting any events in 50 intervals of 30 seconds each. The raw data results are shown in *Tab. 2*. We also plotted the occurrence of each specific count, also known as a histogram, in *Fig. 2* below.

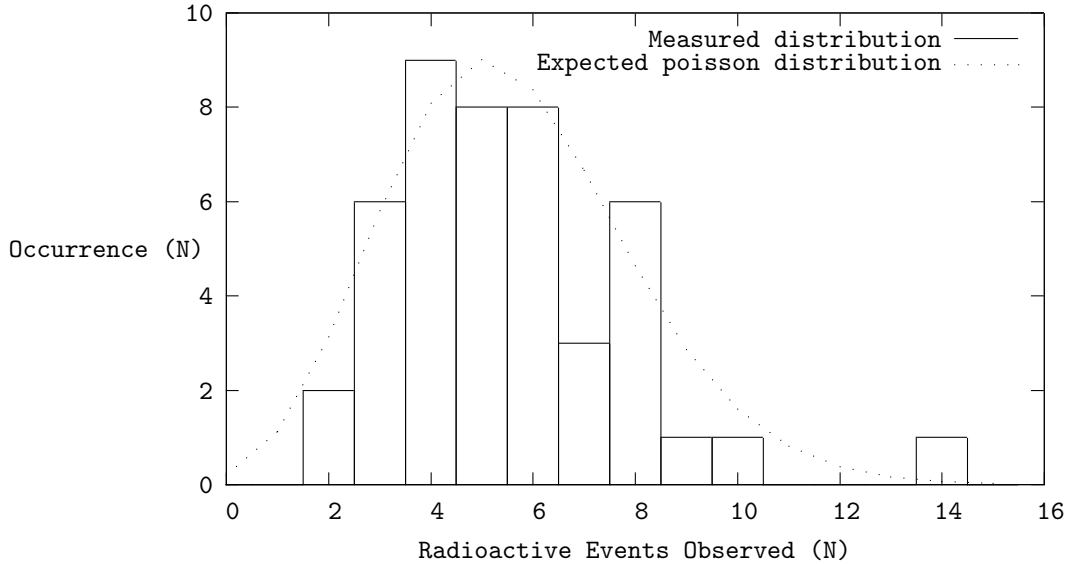


Figure 3: Plot of occurrence of a specific count of radioactive events. The overlaid line graph illustrates the expected distribution as calculated using Poisson with $\mu = 5.57$.

Although background radiation is random, we can attempt modeling it using a Poisson distribution. In our case, we worked with our 50 samples, assuming $\mu = \bar{N} = 5.57$. We plotted the distribution using the following equation:

$$O(N) = 50 \frac{\mu^N e^{-\mu}}{N!} \quad (1)$$

Where $O(N)$ describes the number of expected occurrences of an event count of N . As we can observe from *Fig. 2*, our observed distribution from the lab environment moderately fits our expected Poisson distribution.

4.3 Counting Statistics

Third, we measured the radioactive behavior of a ^{60}Co sample. We counted events for 25 intervals of 1 minute each. The raw data results are shown in *Tab. 3* above. We then calculated the average:

$$\bar{N} = \frac{1}{2} \sum_i N_i = \boxed{352} \quad (2)$$

Then we calculated the sample variance. The values for $(N_i - \bar{N})^2$ are shown in *Tab. 3*, allowing us to quickly calculate s . When we compare it with the quick estimate $\sigma = \sqrt{\bar{N}} = 18.76$, s seems very reasonable.

$$s = \sqrt{\frac{1}{n-1} \sum_i (N_i - \bar{N})^2} = \boxed{19.65} \quad (3)$$

Using $\sigma = s$ and examining all data collected, 72% of all intervals fall within ± 1 standard deviation, and 96% fall within ± 2 standard deviations. Considering Gaussian statistics, where 68% fall within ± 1 standard deviation and 95% fall within ± 2 standard deviations, the quality our data set is superb.

$$\frac{\sigma}{\mu} = \frac{19.65}{352} = \boxed{5.58\%} \quad (4)$$

Using σ and μ , we also calculated the uncertainty in our counting rates. To reduce this uncertainty to 0.1%, we would need to increase our time about 56-fold, or to about 28 minutes.

4.4 Deadtime Determination

Finally, we investigated the dead-time inherent with the detector. This is important because we miss counting any events that occur while the detector is recovering from a recent event. We counted the events for three conditions (1) only sample A, (2) only sample B, and (3) both samples together. A time period of four hours was used. The actual results are listed in *Tab. 4* above.

$$R_{(1)} + R_{(2)} = R_{(3)} \quad (5)$$

$$r_{(1)} + r_{(2)} = r_{(3)} + r_{miss} \quad (6)$$

Ideally, the count rates from (1) and (2) should total the count rate from (3). However, deadtime causes the combined rate (3) to be lower. We then use the missing counts to estimate the deadtime. First, we know that when both samples are placed together, the counts should be equal to the sum of both individual counts, illustrated by R_i terms. However, the actual measured rates, illustrated by r_i , have a rate of missed counts. Rates are calculated from *Tab. 4* and are expressed in Count (N) per second.

$$T_d = \frac{r_{(1)} + r_{(2)} - r_{(3)}}{2r_{(1)}r_{(2)}} \quad (7)$$

$$T_d = \frac{(7.04 \text{ N/s}) + (0.53 \text{ N/s}) - (7.52 \text{ N/s})}{2(7.04 \text{ N/s})(0.53 \text{ N/s})} = \boxed{6.7ms} \quad (8)$$

When thinking about the limitations of our determination of deadtime, we clearly know that our counts are coming from one set of long data samples. In the earlier parts of the lab, we collected several sets of data, allowing us to better approximate values. However, because this part of the lab was so time consuming (12 hours for each run of the three cases), we were limited to just this one run.

The error involved in determining the deadtime is actually an accumulation of error from the three measurements used in its calculation. We know that the error for one each measurement is $\sigma = \sqrt{N}/T$, which has been calculated and listed in *Tab. 4* above.

$$\sigma_{T_d} = \sqrt{\sigma_{(1)}^2 + \sigma_{(2)}^2 + \sigma_{(3)}^2} = \boxed{3.24\%} \quad (9)$$

Finally, when we think about background radiation, we can remember that $r_{back} = 0.186 \text{ N/s}$ was found earlier. We can correct for this rate by expanding it to four hours and assume that $N_{back} = 2678.4$. The values corrected using this approximation are shown in *Tab. 4* above.

5 Conclusion

For this lab, we first found the voltage threshold of the Geiger detector to be $V = 320$ volts. For our measurements, we chose to use $V = 500$ volts so that we would be reasonably far from the threshold, but still on the plateau.

Removing all samples from the area, we found the background radiation to be 0.186 N/s on average. We showed that our multiple measurements followed a Poisson distribution of randomness centered around that average.

After working with one sample, we found the uncertainty of our measurements to be 5.58%. This was using measurement intervals of 30 seconds. By increasing the interval to 28 minutes, we estimate the uncertainty would have been reduced to 0.1%.

Finally, we used various configurations of two samples to estimate the deadtime inherent with our detector setup. It was estimated to be $6.7ms$, with an error of 3.24%.

Before beginning the lab, we formed expectations about what the lab results would look like. All of our measurements and calculations produced those expected results.